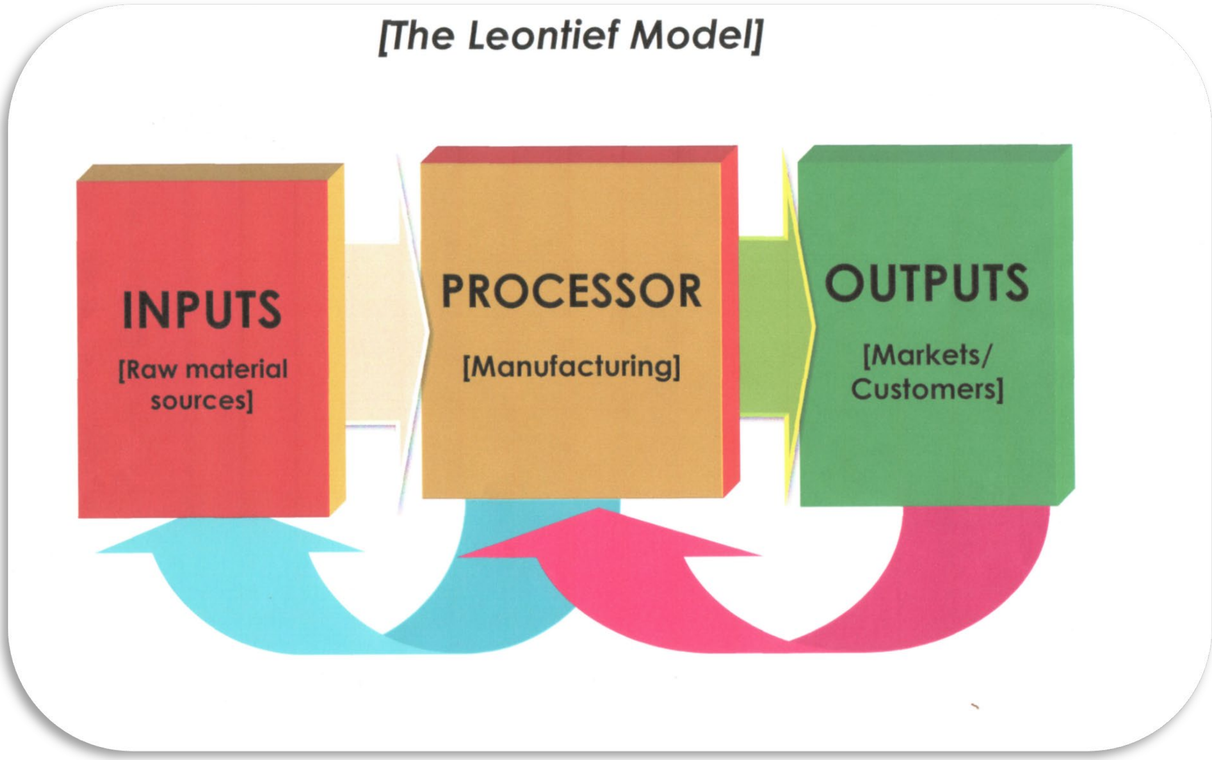
Unbalanced Caribbean Economic Growth in a Dynamic Leontief Model

Perspectives from the Plantation Economy Model



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October 10, 2023

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Development Essays, Issue 1, No. 4.

UNBALANCED CARIBBEAN ECONOMIC GROWTH IN A DYNAMIC LEONTIEF FRAMEWORK: PERSPECTIVES FROM THE PLANTATION ECONOMY MODEL

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20/02/2023

Abstract

This paper uses the Dynamic Input-Output Framework of Leontief (1953; 1970) to represent and explain unbalanced growth in a Caribbean economy along the lines first proposed by Best in his plantation economy model of 1968 and sought in Best and Levitt (2009) and Best and St Cyr (2012).

Tapia House Movement, Maracas, St. Joseph. Managing Editor, Lloyd Taylor.

INTRODUCTION

Economists are generally concerned with growing the living standards of society through the strategic factors of structural change, institutional development, and innovation. Investment to improve these strategic factors also drives growth by increasing augmented labour productivity growth, growth of knowledge, skills and self-confidence, and growth of employment relative to the cost of augmented labour under favourable factor market conditions. In import-dependent economies, such investments are also normally self-sustaining because the productivity growth they induce leads to validating growth of savings and growth of exports to cover the imports on which they depend.

The productivity growth process is generally understood to be multi-industrial with characteristic interdependencies that are important to the adequate representation of how aggregated endogenous growth occurs. Industries were clustered conveniently for analytical purposes into several sectors by Lewis (1954), Best (1968), and Best and Levitt (1969). Lewis (1954) formulated an endogenous growth model with sectoral interdependencies applicable to a surplus labour economy, with a specific mechanism to explain how the long-run growth rate is determined by investment that generates validating savings adjustment and rationalisation of exports by capitalists operating within the system. In the case of Best (1968) and Best and Levitt (1969), appropriate row operations on, and other extensions of, the static multi-industry Leontief model underlies the innovative representations of plantation economy with an import-dependent plantation sector whose growth is constrained by such dependencies and a residentiary sector whose growth is accelerated in the long-run by increasing reliance on the production and accumulation of domestic capital, especially knowledge, skills, and self-confidence. However, the static framework was formulated as an open system, with significant exogenous final demand and value-added variables, and with no closure that makes endogenous variables out of investment and savings, possible structural change, and growth. Output is endogenous to investment, but not vice versa. In other words, while an exogenous change in investment demand or value-added is guaranteed to force a responsive change in output, factor demand, and prices, there is no specific mechanism by which a change in output or factor demand could assuredly induce a sufficient responsive change in investment or any of the other elements of final demand. Similarly, while an exogenous change in factor prices or value-added (factor demand) would drive a responsive change in output price, there is no mechanism by which an adjustment of output price would induce a sufficiently responsive (or validating) change in factor demand and factor prices, especially the rate of profit and savings or the rate of exports, that motivate the investment.

This paper uses the dynamic Leontief framework as a reference point to offer some updates on the type of closure that fills that gap and accounts for unbalanced growth along the lines predicted by Best (1968) and Best and Levitt (1969). In particular, it provides specific methods characteristic of the economy and the nature of its factor markets by which each sector purchases in each time period enough capital goods to assure adequate capacity for its own current and anticipated future production; and by which each capital-producing sector produces enough capital goods and services to satisfy its own needs and meet current investment orders from other sectors for capacity to be established to meet their needs for current and future productive capacity. It is demonstrated that addressing the endogeneity of growth involves the identification of a specific mechanism for determining the rate of profit on the capital stock and the associated rate of savings adequate to cover the investment in additional final capital inputs. In that mechanism, there is room for product market power enjoyed by capitalists on the one hand and for labour market power enjoyed by workers and managers with knowledge, skills, and self-confidence. Similarly, it is shown that, in an import-dependent economy, the endogeneity of growth involves a mechanism for growing exports to validate the imports needed by the investment process.

THE STATIC LEONTIEF MODEL

Let M be a matrix of direct (interindustry) input coefficients expressing intermediate demand for the supply vector of industry outputs (x), F be a vector of total final demands for industry outputs, i.e., the sum of consumption (C), investment (I), government spending (G), and net exports $(X - J)$ that output can support with the claims on value-added that is generated through production. The coefficients $m_{ij} \geq 0$ of M represents the amount of output sold by industry i to industry j in a given production year, divided by the total output of industry j. What is considered the total output depends on the unit in which the supply of industry i is measured. If it is measured in physical units uniquely appropriate to itself, then the appropriate total output is the row total. The same is true if the units are mixed, some physical and some monetary. On the other hand, if all the supplies are measured in monetary units, as is the typical case, the column total would be appropriate.

Also, let L be the $k * n$ matrix of coefficients of factor inputs and import from the value-added and import rows of the data matrix. Each row of L contains the unit requirements of a factor in the production of the output of each sector j. In the case of capital, the input requirements are the current services of the stock of capital or consumption of fixed capital. In the case of the row vector of augmented labour, for example, l_j is the level of worker effort per unit of the total output of industry j. Worker effort is a product of knowledge and skills per worker and the number of workers needed per unit of output. When the coefficients of the j^{th} column of M are supplemented by the jth column of factor inputs (from the value-added rows) l_j, the complete column indicates all the inputs needed for industry j to produce a unit of output. A column of technical coefficients (corresponding column of M and L) describes the average technology in use in industry j. Every industry is assumed to produce a single characteristic output using a single average or optimal technology. The coefficients of M and L are all expressed in basic price[1] values per unit of industry output, are assumed to represent the most efficient technologies available, and are therefore assumed fixed in the sense of changing over a longer time scale, even if F varies with t.

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[1] In the SNA, a basic price is the amount a producer receives from a purchaser per unit of goods or services produced, minus the taxes on the products and plus any subsidies on the products.

Further, let p be a column vector of the prices of x, expressed in money units per unit of output, and w be the column vector of the factor prices (in money units per input unit) of the factors L measured in physical units. So, the wage rate is expressed in money unit paid per worker or per worker hour, capital services are priced at a depreciation allocated per unit of capital stock per period or the depreciation rate, and the like. Also, let v be the column vector of total value-added, the vector of the sums of the payments to the factors of production in L. It must hold that $v' = w'L$. This is also a measure of the financial capital required to transform intermediate inputs into output- the sum of the wages, depreciated machinery, and profits (including surpluses of government enterprises), plus taxes. It is assumed that the price of each industry's product is the sum of the payments to a unit of each unproduced factor, so $p'(I - M) = v' = w'L$.

The static Leontief model is a set of quantity and price equations:

1. $x(t) = (I - M)^{-1}F(t)$
2. $l(t) = Lx(t)$
3. $p' = v'(I - M)^{-1} = w'L(I - M)^{-1}$
4. $p'F(t) = v'x(t) = w'Lx(t)$

The quantity equations (1) and (2) are recursive, as are equations (3) and (4). So, the outputs are first derived from equation (1) and then used to determine the levels of input demand. Equation (3) satisfies the condition that price is unit value-added, augmented by the long-run multipliers of the economy. Equation (4) is an identity that follows from the pre-multiplication of equation (1) by p' and using equation (3) with the condition that $p'(I - M) = v' = w'L$. It shows that the value of final demand is equal to total value-added, which is the value of all factor inputs. Here too, the prices are first derived with equation (3) and then used to establish the identity in equation (4).

In equations (1) and (2), goods and services ($x(t)$) are distinguished from resource inputs ($l(t)$) and final demand. In equations (3) and (4), prices of goods and services are distinguished from their quantities and prices of goods and services are also distinguished from the prices of factor inputs. It should be noted that in practical work, the supply and use tables, national accounts, and input-output tables underlying M and L are as important as equations (1) to (4). Given M and L, equations (1) and (2) can be used to consider the effects on x and l of alternative hypotheses about variations in $F(t)$. Equation (3) can be used to determine the effects on the price of alternative hypotheses about variations in factor prices. In equation (1), the idea is represented that the economy must produce a greater level of output than the stimulating vector of final demand. The matrix $(I - M)^{-1}$ is conventionally referred to as the analytical *total production requirements matrix,* a matrix of long-run multipliers that represents the direct and indirect requirements of industry output to satisfy a specified level of final uses. In economic terms, while an element of M describes the relationship between two industries, $(I - M)^{-1}$ summarises the interdependencies of all industries and hence the backward and forward linkages of an economy, i.e., the long-run intermediate demand by each industry for the outputs of all other industries and the long-run intermediate supply of the output of each industry to all others for their use in production to satisfy the level of final demand. In equation (2) the matrix of total factor employment is similarly represented as a result of final demand multiplied by the product of the matrix of factor coefficients and the matrix of total production requirements. Similarly, in equation (3) the prices of goods and services are represented as the multiple of value-added and the matrix of total product requirements. The system in (1) to (4) is a linear system, which relates exogenous final demand to total industry output and to factor (including import) demands, as well as to price, via the matrix of inter-industry linkages and the coefficient matrix of factor demands.

For an economic interpretation, the elements of the supply vector (x), the price vector (p) and the matrices L must be non-negative, given non-negative F and non-negative v, and $(I-M)^{-1}$ must be strictly positive. The latter condition is normally considered from a variety of perspectives. First, it is known that $(I-M)^{-1} = I + M + M^2 + \cdots$, with $\sum M^k$ converging to some positive finite value as $k \to \infty$. This implies that if $(I-M)^{-1} > 0$, the Hawkins-Simon condition (Hawkins and Simon, 1949) must hold, which indicates that if M is irreducible, then all the (successive or leading) principal minors of $(I-M)$ (and therefore $(I-M)^{-1}$), are also positive. This implies that each subgroup of industries demands less input from the economy than the value of the output it produces. So, if M is constructed using monetary values, then the Brauer-Solow condition holds that value-added in each sector (the money left over to cover or validate financing of factor inputs) is positive because a choice of units can be made such that all row sums or all column sums of M are smaller than unity (Solow, 1952). Considering equation (1), post-multiplication by F gives $x = F + MF + M(MF) + \cdots$, which describes the successive rounds of supply requirements that must be met in the attempt to satisfy the exogenous demand F. That is, the economy must produce F, and must also produce the intermediate inputs needed to produce F, as well as the intermediate inputs needed to produce those intermediate inputs, and so on, in a convergent process summarized by $(I-M)^{-1}F$.

From the perspective of characteristic values and vectors,[2] it must hold that M has a dominant characteristic value, λ, with $0 < \lambda \leq 1$, which measures the size of the intermediate outputs of the economy relative to total output, and hence to surplus or value-added. This characteristic value bears an inverse relation to productivity. So, the smaller is λ, the higher is factor productivity and the larger is the surplus available for payment of claims by owners of the factor inputs; i.e., to wages, profits and interest to local or foreign owners of stock, taxes, and the like. The distribution among claimants is a matter of comparative market power. However, in general, the claims paid can be used for the elements of final demand,

[2] Recall that for any matrix M, a characteristic value λ is a solution of $|M - \lambda I| = 0$, where the left-hand side of the equation is a characteristic polynomial, whose degree is the order of M. By the fundamental theorem of algebra, this also implies that $|M - \lambda I| = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \ldots (\lambda_n - \lambda)$. The numbers λ_i, which may be complex and not all distinct, are roots of the polynomial and the characteristic values of M. If M is either positive or negative, then by the Perron-Frobenius theorem, there exists a characteristic value that is uniquely the largest of these, usually called the dominant characteristic value (Minc, 1988).

consumption, reinvestment, government spending, and the like. Further, λ increases if any element of M increases and decreases if any element of M decreases. So, for example, if there is a technological innovation that causes some m_{ij} to become smaller, then λ decreases while at least one factor of productivity grows, and the available surplus also grows. These characteristic values play an important role in the dynamic models below as indicators of the behaviour of growth rates and profit rates. In some extreme cases, an economy might be unable to produce a surplus to finance its final demand, so it would be necessary to set $F = 0$ in equation (1). In such a case, $x = Mx$, the equality in the above restriction applies, and the maximum characteristic value is $\lambda = 1$. Then, the solution x is the Perron-Frobenius characteristic vector. The resulting vector x would identify only the structure of the economy and its scale would have to be identified with additional information.

While the model in equations (1) to (4) assumes an average technology, it can discriminate between any two or more technologies, by identifying the one among them that is the least cost and most efficient technology. Thus, suppose the columns of another pair of matrices M^* and L^* were to describe an alternative technology to that in M and L, One way to compare them would be to compare the maximum characteristic λs of M and M^*. The one with the lower λ would be the more productive technology, even if it had some other undesirable features. Further, the resulting costs of factor use, $w'Lx(t)$ and $w'L^*x(t)^*$, described by the equations (1) to (4) could be compared to determine the one that is cheapest in terms of overall factor costs for any given $F(t)$. In principle, it should turn out that, for a given $F(t)$, the one with the smaller maximum λ would also be more efficient and yield the greater amount of surplus and lowest prices to support its direct and indirect use in production. This would hold because some elements of the M matrix and all elements of the multiplier $(I - M)^{-1}$ are appropriately smaller.

In this static Leontief model, there is no reference to the rate of growth, or to institutional progress and innovation, which are key variables in growing the living standards of society. Investment which drives the rate of growth is fully represented in the given matrix of final demand, alongside public and private consumption spending, and exports. Thus, investment merely documents exogenous additions to the stocks of fixed capital items such as buildings, machinery, tools etc., and how these are satisfied by domestic production. In a satellite setting, it might also include additions to human capital, especially the knowledge, skills, and self-confidence of workers. Similarly, factor demand and related rates of return on investment are also fully represented in the value-added matrix, providing exogenous stimulus to endogenous price formation, without indicating how changes in prices would induce adjustments in value-added, and in particular the rate of profit.

However, in a dynamic representation of the economy such as is needed to represent, structural change, growth and the forward-looking development planning process, investment and value-added cannot be treated this way. There must be a way to guarantee that changes in output induce the relevant investment and (hence) growth response as well as to guarantee that changes in product prices would cause suitable changes in factor costs, especially the wage rate and the rate of profit, to induce the required investment response. That is, investment, and hence long-run growth, must be explained within the model of production and this must also be done in a way that allows accounting for the nature of the factor markets, productivity growth, profits and savings resulting from the optimizing investment behaviour of agents or from the actions of some planner or policy maker seeking to maximize some objective function, or both. All of this must be done simultaneously at the economy-wide and industry levels. This is ultimately what a dynamic Leontief model can be adjusted to achieve, by introducing a suitable closure which assures that, in each t, each capital-producing industry produces enough final capital inputs to meet the required capacity for its planned future production and also enough final capital to satisfy the current investment orders by other industries to meet their future requirements of added capacity.

INVESTMENT IN THE DYNAMIC LEONTIEF MODEL

We have been reminded since Lewis (1954) that the principal mechanism for achieving structural change and growth is investment in final capital, especially those that embody new technologies and are produced in the economy. Such investment can be made endogenous in a dynamic Leontief model. The key to a dynamic Leontief model with endogenous investment is that a unified inter-industry matrix can be constructed in which each industry is represented as supplying intermediate capital, final capital, or both, to meet current and future capital stock requirements in the light of existing technology. Moreover, in the associated price system, profits and savings can be identified that validate the investment in expanded capacity. In that framework, an industry's current investment is assumed to call for a variety of goods and services to be produced at some time t by other industries to add to the former's production capacity with at least one lag relative to the period in which the capacity will be used. A dynamic input-output model keeps track of both the intertemporal and inter-sectoral relationships and assures their consistency. The overall approach allows additions to the stocks of durable capital goods to be technologically required, given the technique in use, so that an expansion of productive capacity matches the rate of growth of the level of output for which there is effective demand captured by the flow of savings from profits.

We start from Leontief's version of the model (Leontief, 1953, 1956, 1970), which has the form of a fundamental difference equation of output:

5. $x(t) = Mx(t) + B[x(t+1) - x(t)] + F(t)$

where M is the Leontief inter-industry matrix of coefficients, $x(t)$ is an n-vector of output levels and $F(t)$ is an n-vector of final demands, excluding investment and imports. The matrix B is the matrix of (stock) coefficients of final (fixed) capital stocks, including human capital, with elements $b_{ij} = \frac{f_{ij}}{x_j} \geq 0$ that describes the amount of capital output supplied by industry i to industry j (f_{ij}) to be held as capital stock for production of a unit of its output, without considering inventories of intermediates held over. This means that $x(t)$ includes a set of "human capital-producing industries", that produce as commodities that can be accumulated the knowledge, skills and self-confidence, as well as other forms of human capital, of workers through commodities (intermediate and final capital) and labour. In the context of a labour-surplus economy, it is assumed that the underlying condition of capital scarcity in key sectors implies that capital is distributed among the existing capacity in such a way that it is fully utilized and that demand for investment to eliminate surplus labour makes it unnecessary to address the sectoral rates of capacity utilization explicitly. In other words, $x(t+1) - x(t)$ is based on projections that reflect past rates of growth under conditions of full capacity utilization. The assumption also means that output can be deduced from the system-wide matrix inverse and its multipliers and interpreted in terms of its eigenvalues.

It can be assumed that the economy operates over the period $t = 0,1 \ldots N-1$, looking ahead to period N. Then, if x is an n-vector of production and $F(t)$ a given vector of final demands, equation (5) is a set of nN equations in $n(N+1)$ unknowns with solutions that represent a Leontief trajectory of the economy over $0,1,2, \ldots N-1$. The fundamental question is whether equation (5) is a meaningful representation of how current output is affected by future planned investment and can accommodate accounting for an endogenous long-run growth rate that is informed by consideration of the nature of the factor markets, productivity growth, profits, and savings.

After considering problems with the existence of B^{-1}, since all sectors do not produce final capital, Leontief's basic approach to solving for $x(t)$ (Leontief, 1970) was to write (5) as

6. $[I - M + B]x(t) = Bx(t+1) + F(t)$

Equation (6) says that, considering current interindustry and investment coefficients, current output, including the supply of intermediate and final investment demand, is shaped by future planned investment demand and current final demand, i.e., consumption, government spending and exports. Then, if, as is likely, $[I - M + B]$ is non-singular, we can write:

7. $x(t) = [I - M + B]^{-1}\{Bx(t+1) + F(t)\}$

The solution in equation (7) is a backward recursion and is logically reasonable only when $x(t+1)$, and hence any terminal position $x(N)$, for $N = (t+1)$, is treated as a goal that guides the current normal goal-oriented decision-making processes of finitely living social agents. It is also not in the standard form of a difference equation for the application of theorems related to stability. The importance of such considerations is magnified when the agents of the economy must account adequately for import dependence and the associated demand for foreign exchange. However, it cannot be assumed that $I - M + B$ satisfies the requirement that the economy produces a surplus to fund $F(t)$ and $[I - M + B]^{-1}$ does not have a natural interpretation as an analytical *total production requirements matrix* that represents the direct and indirect requirements of industry intermediate and final capital output to satisfy a specified level of final uses. Further, the solution masks the role of the rate of growth of the economy, and for critical influences on such growth of forces such as government policies, the state of the factor markets and related factor prices, product prices, productivity growth and profit growth associated with investment, and consequences for the flow of savings. The approach set out below explicitly addresses these and other challenges related to the structure of specific types of economies.

INTRODUCING GROWTH AND PROFITS IN A TRADING CONTEXT

The fundamental difference equation of output becomes especially interesting when it is viewed as specifying the level of activity required today to enable the achievement of some target $x(t+1)$ in the near term that allows for growth, given M and the current level of final demand $F(t)$ (consumption, government, and exports). However, this is simply an alternative way of specifying a desired common general growth rate across all industries in year t, $g_x(t)$, such that $x(t+1) - x(t) = g_x(t)x(t),\ with\ g_x(t) = \frac{dx(t)}{x(t)}$. Thus, by definition,

8. $x(t+1) = \big(1 + g_x(t)\big)x(t).$

If $g_x(t) > 0, \forall t$, then there is growth, and specifically, there is balanced growth; and if $g_x(t) < 0, \forall t$, then there is economic decay.

Using (8) in (5), and replacing the term $B[x(t+1) - x(t)]$ with $Mg_x(t)x(t)$, allows the definition of the adjusted productive capacity that must be constructed in t to support the production of $x(t+1)$, using output that must be produced or imported as part of $x(t)$. Thus, the fundamental equation takes the form $x(t) = Mx(t) + g_x(t)Mx(t) + F(t)$ and the resulting model with an explicit desired growth rate becomes:

9. $[I - (M(1 + g_x(t))]x(t) = F(t)$

Ultimately, therefore, in contrast to equation (5), we must work with an augmented matrix $M(1 + g_x(t))$ and a problem with $g_x(t)$ to be determined for each period. Thus, for each t there are now n equations with $n+1$ unknowns, including $g_x(t)$, which is treated as common to all industries. Then, over $t \subset \{0,1 \dots N-1\}$, there are nN equations in $N(n+1)$ unknows. Now, the elements $m_{ij}(1 + g_x(t))$ of the augmented matrix $M(1 + g_x(t))$ describe the total intermediate and additional final capital supplied in the current period by industry i to industry j for the production of a unit of its output in the current period, and for installation of the new capacity that must be added to enable the production of a unit of its output in the next period. This allows for the fact that some industries do not produce capital supplies. It is in this sense that in period t a capital-producing industry produces enough final capital to assure adequate capacity for its own anticipated future production and also satisfy the current investment orders of other industries for final capital to add to their capacity.

From the perspective of the development challenges of the economy, a crucial consideration introduced by Best (1968) was the structure of domestic production in terms of the number and relative size of industries that specialize in producing semi-processed intermediate exports (muscovado bias) and that are highly dependent on imports for their capital stock. Here, we capture that concept by identifying industries that produce and distribute final capital to other domestic industries and to exports and industries that supply non-competitive intermediate and final capital imports to other industries. Competing imports are assumed to be included in the column vectors of final demand. Together, these approaches indicate the degree of dependence of various industries and final users on imports, especially final capital inputs.

Table 1 represents the summary system considerations. The structure of M is defined to include capital production. Thus, the n using industries would demand domestic supplies of intermediate and final capital imports, with competitive imports showing up in final demand. It is noted that the fewer the number of industries in $x(t)$ that produce and distribute final domestic capital inputs, the smaller their share in total output; and the greater the share of industries that must import their final capital stock, the greater will be the development challenges of the economy (James and Hamilton, 2022). On this basis, the augmented coefficients $m_{ij}(1 + g_x(t))$ of $M(1 + g_x(t))$ represent the amount of intermediate and/or final capital output sold by industry i to industry j in a given production year, divided by the total output of industry j, where that total output includes the value of its capital installation activity. The suppliers of competing imports are assumed to be among the n industries that produce domestic intermediates and/or final capital. The rectangular matrix of total non-competing imports, Z, is included under the n columns of intermediate demand, above the rectangular matrix of non-produced resource inputs that contribute to value-added, (L). The rectangular resource matrix L still identifies the financial capital needed to use intermediates and final capital in production. Thus, the entries in the jth column of $M(1 + g_x(t))$ includes all domestic output needed for industry j to produce a unit of joint output. When this is supplemented by the jth column of the rectangular matrix of non-competing import coefficients $Z = \begin{bmatrix} Z^m \\ g_x(t)Z^c \end{bmatrix}$, with Z^m representing rows of intermediate imports and Z^c rows of final capital imports, and the jth column of factor input coefficients (from the value-added rows of the rectangular matrix L), the column indicates all the inputs (and financing) needed for industry j to produce a unit of output. A column of technical coefficients describes the average technology, including imported inputs, used in industry j. The implied assumption is that every industry produces a single characteristic output combination of intermediate and final capital, including own-capital installation, using a single average technology.

Table 1: Structure of a National Symmetrical Input-Output Table with Endogenous Investment

		Sectors j				Sectors j using final capital							
		1	2		n	1	2		n	C	G	X	Jcomp
Sectors j	1	Intermediate Demand (M)				Final capital demand ($(g_x(t)$M)							
	2												
	n												
Non-competitive importers	J1	x	x	x	x								
	J2	x	x	x	x								
	Jc1					X	x	x	X				
	Jc2					x	X	x	x				
Value-added (financial capital needs)	L1	l	l	l	l								
	L2	l	l	l	l								
	L3	l	l	l	l								
Total													

For now, the matrix M is assumed to be time invariant but reflects the institutional and technological arrangements of industries at the time of SUT compilation (UN, 2018: 406). That is, even if M varies, this occurs on a longer time scale than the t over which F or x vary. The term $Mx(t+1)$ defines capacity that must be in place to support projected production of $x(t+1)$. So, the difference term $g_x(t)Mx(t)$ allows for an expansion of productive capacity during period t that matches the expansion in the level of intermediate and final capital output effectively demanded in the light of the stocks of durable capital goods that are technologically and institutionally required, given the nature of the institutions and the technique in use as documented by M.

To understand the nature of $g_x(t)$ more fully, we return to equation (9) and assume that $F(t) = 0$. Then, subject to the restriction that $g_x(t) \neq 0$, we have:

10. $Mx(t) = \frac{1}{(1+g_x(t))}x(t)$

Equation (10) is a characteristic value equation and $\frac{1}{(1+g_x(t))}$ is the characteristic value (eigenvalue) that magnifies or contracts $x(t)$. Further, since $x(t) > 0$, and defining the Perron-Frobenius or dominant eigenvalue of M as μ, we get:

11. $\frac{1}{1+g_x(t)} = \mu \; or \frac{1}{\mu} = 1 + g_x(t)$

Thus, the rate of growth in equation (9) is technologically determined from the maximum eigenvalue of M.

The term $M(1+g_x(t))$ in equation (9) represents an extended domestic inter-industry system, the sum of demand for intermediate capital output to support production in any t and demand for final capital output to add capacity to support production in $t+1$. Thus, (9) yields a solution for $x(t)$, for factor demand $l(t)$, as well as for necessary non-competitive imports, $z(t)$, as:

12. $x(t) = [I - M(1+g_x(t))]^{-1}F(t)$
13. $l(t) = Lx(t)$
14. $z(t) = \begin{bmatrix} Z^m \\ g_x(t)Z^c \end{bmatrix} x(t)$

Equations (8), and (12) to (14) now define a forward-looking recursive process. Here, equations (13) and (14) indicate that the resources that must be financed to enable domestic production include the necessary non-competing imports, which are essentially factors of production whose productivity matters in the domestic economy. Equation (13) embeds the underlying dependence of the production system on resource financing capacity, and it allows the availability of surplus labour by assuming that whatever labour requirements are implied by $x(t)$ would be available in the local market. Equation (14) cannot escape the problem of financing capacity since it treats imports as necessary unproduced non-competing resources and affirms the dependence of import needs on the growth rate, $g_x(t)$. Feasibility cannot be assured, since the sum of exports and inflows of foreign exchange might not cover the imports implied by the equation. The development of capacity to generate sufficient exports to ensure self-sustaining growth is a persistent challenge of this type of economy.

In equation (12), the multipliers of the economy can now be quantified as the outcome of an analysis. The term $[I - M(1+g_x(t))]^{-1}$ now has the characteristics of an analytical *total production requirements matrix* that represents the long-run direct and indirect requirements of industry output of intermediate and final capital output to satisfy a specified subset of final uses, consumption, government spending, and exports, taking into account the current rate of growth of the industries to meet the capital requirements of future production. In economic terms, $[I - M(1+g_x(t))]^{-1}$ summarises the full set of backward and forward linkages of an economy, i.e., the long-run multipliers that now include the investment multipliers. That is, equation (12) can also be treated as a specification of the successive rounds of intermediate and final capital supply requirements that must be met in the attempt to produce the exogenous F. That is, the economy must produce F, along with the intermediate and final capital inputs $M(1+g_x(t))F$ needed to produce F, as well as the intermediate and final capital inputs $(M(1+g_x(t)))^2F$ needed to produce those intermediate and final capital inputs, and so on; a process which converges to $[I - M(1+g_x(t))]^{-1}$ when $(M(1+g_x(t)))^k \rightarrow 0 \; as \; k \rightarrow \infty$.

Here too, for an economic interpretation, the elements of $[I - M(1 + g_x(t))]^{-1}$ must be strictly positive, given non-negative x and F. This implies that there is some maximum value of $g_x(t)$ beyond which $[I - M(1 + g_x(t))]^{-1}$ can be negative. Once below this threshold, the solution in (12) still characterises how output of $x(t)$ is determined by a subset of final demands $F(t)$ mediated by these total intermediate and final capital production requirements. If $(I - M(1 + g_x(t)))^{-1} > 0$, the Hawkins-Simon condition (Hawkins and Simon, 1949) must also hold, and therefore each subgroup of industries must demand less (intermediate and final capital) inputs from the economy than the value of the output it produces. Moreover, the economy must produce F, and must also supply the intermediate and final capital inputs needed to produce F, as well as the intermediate and final inputs needed to produce those intermediate inputs, and so on, all sufficient to meet future production needs, in a convergent process summarized by $[I - M(1 + g_x(t))]^{-1}F$.

Further, the perspective of characteristic values of the economy can also be represented as the outcome of analysis. A necessary and sufficient condition for $\left(I - M\left(1 + g_x(t)\right)\right)^{-1} > 0$ is that M has a maximum characteristic value, $\frac{1}{1+g_x(t)}$, with $0 < \frac{1}{1+g_x(t)} < 1$, which in this case measures the size of the intermediate and final capital outputs of the economy relative to total output, and hence to surplus over the costs of intermediate and final capital inputs. This characteristic value, $\frac{1}{1+g_x(t)}$, converges on 1 as $F \to 0$ but still bears an inverse relation to productivity, in that the smaller is λ and hence the greater is $g_x(t)$, the higher is factor productivity and the larger is the surplus available for payment of claims by owners of the factor inputs; i.e., to wages, profits, and interest to local or foreign owners of stock, finance, taxes, and the like. As before, the claims paid can be used for the elements of final demand, consumption, reinvestment to build up capital stock, government spending, and the like. Further, λ increases if any element of M increases and decreases if any element of M decreases. And, it still holds that a technological innovation that causes some m_{ij} to become smaller will also decrease λ so that at least one factor of productivity would grow, and the available surplus for distribution would also grow. These characteristic values necessarily play an important role in restricting the behaviour of growth rates and profit rates. In particular, $g_x(t)$ is restricted to values such that $\frac{1}{1+g_x(t)}$, the dominant eigenvalue of M, is less than 1.

With respect to $x(t)$, the growth rate $g_x(t)$ is an endogenous but technologically determined variable in equation (12). First, as an eigenvector of M with positive values, $x(t)$ depends on $g_x(t)$ because it determines the scale of current output required to grow the capacity needed for production in $t + 1$. Second, $g_x(t)$ is enabled by $x(t)$, including the expansion of production required to enable installation of necessary capacity for production in $t + 1$. And, in particular, from equation (10), with pre-multiplication by x^*, $\frac{1}{1+g_x(t)}$ is determined as the Rayleigh Quotient:

15. $\frac{1}{1+g_x(t)} = \frac{x^*Mx}{x^*x}$

where x^* denotes the transpose of x. It would also hold that:

16. $\frac{1}{(1-\lambda)} = \frac{x^*[I-[(M(1+g_x(t)))]^{-1}x}{x^*x}$

Thus, λ also depends on $g_x(t)$. It follows that for each t there are still n equations with $n+1$ unknowns, including $g_x(t)$, since identification of $x(t)$ in equation (12) requires additional information that can be used to determine $g_x(t)$ than is available in from equations (15) and (16) as they stand. The principle applies to all periods in the trajectory, i.e., to all $t \subset \{0,1,2,\dots N-1\}$. The direction in which to look for the additional information is in the associated price model.

Regarding prices, it is necessary that current prices now cover the cost of intermediates plus the price increases, $[p'(t+1)-p'(t)]B$, needed to induce investment in new capacity, plus the factor costs of labour, consumption of fixed capital in the current period, and the like. Thus,

17. $p' = p'M + [p'(t+1) - p'(t)]B + p'_zZ + w'L$

with p' and w' row vectors of output and resource input prices, and p'_z a row vector of prices for intermediate and final capital imports in domestic currency, with $p_z = \varepsilon p_f$, for ε the exchange rate (the domestic price of foreign currency) and p_f the relevant foreign prices. We assume that current and future prices are related by:

18. $p(t+1) = (1 + s_pr(t))p(t)$

where $s_pr(t) = \frac{dp(t)}{p(t)}$ is the rate of flow of savings from the profits enabled by price changes that are calibrated to cover the cost of increase in the value of capacity installed in period t for use in period $t+1$. It follows that *the* use of (18) in (17) creates the familiar equality of price and unit cost, and gives:

19. $p'\left[I - \left(M + s_pr(t)B\right)\right] = p'_zZ + w'L$

Once again, the assumption of $p'_zZ + w'L = 0$ gives:

20. $p'[I-M]^{-1}B = p'\frac{1}{s_pr(t)}$

So, here too, for $p' > 0$, $\frac{1}{s_p r(t)}$ is the reciprocal of the dominant eigenvalue of $[I - M]^{-1}B$. Since this is identical to $\frac{1}{g_x(t)}$, it yields the important result that:

21. $g_x(t) = s_p r(t)$.

In other words, this "classical assumption" is actually an implication of the model assumptions used so far. If s_p is assumed fixed exogenously, then the additional information required to determine $g_x(t)$ is information needed to determine $r(t)$. We return to this. By the logic of equation (18), the savings flow covers the demands of all capital investment, including consideration of the necessity of price adjustments to cover opportunity costs, and preserve asset valuation, and covers imported capital costs, especially in the event of technical change. Thus, in economic terms, output growth and savings growth are necessarily mutually consistent, in the sense of growing together.

Another way to view equation (21) is that the achievable growth rate is set by the rate of savings from the profits allowed by feasible price adjustments, with the profit rate a free variable to be determined. So, the higher the savings rate, the higher the growth rate achievable. A special case of the price assumption is the Keynesian claim that prices are sticky downwards. However, for given M, there would exist a threshold beyond which $s_p r(t)$ makes $[I - (M + s_p r(t)B)]$ singular, and this also sets an upper limit on $s_p r(t)$ and $g_x(t)$. Subject to the threshold, a key benefit of the result in equation (21) is the mutual consistency and stability of equations (12) and (19). Then, using (21) in (19) gives,

22. $p' = (p'_z Z + w'L)[I - (M + g_x(t)B)]^{-1}$

And,

23. $p'F(t) = (p'_z Z + w'L)x(t)$

The full dynamic Leontief model with endogenous growth and forward recursion is therefore represented by equations (8), (12), (13), (14), (21), (22) and (23). In equation (23), it is asserted that, in the Caribbean case, the value of final demand must be equal to total value-added plus the cost of imported resources, which is the value of all resource inputs financed for production. In equation (22), as in equation (12), the growth rate is an endogenous variable. In each t, p depends on the rate of savings out of profits and the prices determined influence the availability of validating savings, coverage of opportunity costs, and preservation of asset value. This interdependence is made explicit by the known expansion $p' = (p'_z Z + w'L)[I + (M + g_x(t)B) + (M + g_x(t)B)^2 + \cdots + (M + g_x(t)B)^k]$, as $k \to \infty$. So, if Z, L, M, and B are fixed and if Lewis conditions prevail and w is fixed by conditions in the subsistence sector, and p_z is exogenous, then p rises with the prices of imported inputs or $g_x(t)$ and hence $s_p r(t)$. Then, any rise in p necessarily induces a rise in $s_p r(t)$ and $g_x(t)$, at least up to threshold levels if policy interventions are not triggered beforehand. It is also worth observing that under Lewis conditions $w'L$ cannot be zero, so the maximum eigenvalue of $(M + g_x(t)B)$ cannot reach 1, which also sets upper limits on $g_x(t)$. The system must generate surpluses to finance its use of the primary factor inputs and imports.

ISSUES OF STABILITY

An important question is, if $g_x(t)$ is a free variable in the system, what additional information allows for its determination in a way that ensures model stability. To consider this issue, we first introduce a few credible assumptions, to transform the system in (8) and (12) into a standard difference equation form in order to study the sequence $x(t)_{t=0}^{\infty}$ in either qualitative or quantitative terms using the eigenvalues and eigenvectors of the inverse matrix $[I - (M(1 + g_x(t))]^{-1}$. First, we assume that final demand is some fraction α of output, so $F(t) = \alpha x(t)$. It follows immediately that equations (8) and (12) combine to yield the following difference equation as the basic dynamic behaviour of the system:

24. $x(t) = Ax(t-1)$

where $A = \left(1 + g_x(t)\right)\alpha[I - (M(1 + g_x(t))]^{-1}$ is n-dimensional and some initial $x(0)$ can be specified. Equation (24) can be used to represent a recursive growth process.

Now, we can derive an expression for $x(t)$ using the eigenvalues and eigenvectors of A. Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be the eigenvalues of A with corresponding eigenvectors $u_1, u_2, \dots, u_n$. Then, we know that: $Au_1 = \lambda_1 u_1$, $Au_2 = \lambda_2 u_2$, etc. Next, we make the vital assumption that $u_1, u_2, \dots, u_n$ are linearly independent, so $\{u_1, u_2, \dots, u_n\}$ is a basis for R^n. In that case, there must exist some non-zero constants, $b_1, b_2, \dots, b_n$ such that any starting vector $x(0)$ can be represented as:

25. $\mathrm{x}(0) = \mathrm{b_1 u_1} + \mathrm{b_2 u_2} + \cdots + \mathrm{b_n u_n}$

From equation (24) it must hold that

26. $x(1) = Ax(0) = A(\mathrm{b_1 u_1} + \mathrm{b_2 u_2} + \cdots + \mathrm{b_n u_n})$

Or,

27. $x(1) = (\mathrm{b_1} A\mathrm{u_1} + \mathrm{b_2} A\mathrm{u_2} + \cdots + \mathrm{b_n} A\mathrm{u_n}) = \mathrm{b_1}\lambda_1\mathrm{u_1} + \mathrm{b_2}\lambda_2\mathrm{u_2} + \cdots + \mathrm{b_n}\lambda_n\mathrm{u_n}$

By continuous application of equation (24), we get the general expression:

28. $x(t) = Ax(t-1) = \mathrm{b_1}\lambda_1^t\mathrm{u_1} + \mathrm{b_2}\lambda_2^t\mathrm{u_2} + \cdots + \mathrm{b_n}\lambda_n^t\mathrm{u_n}, t = 0 \dots \infty$

Equation (28) provides a formula in terms of the eigenvalues and eigenvectors of A for any sequence $x(t)_{t=0}^{\infty}$ which satisfies equation (24) for $t = 1 \dots \infty$. Equation (28) also provides qualitative information about the solution of the difference equation in (24). For example, if

$[\lambda_i] > 1$, then if $x(0) \neq 0$, the sequence $x(t)_{t=0}^{\infty} \to \infty$ and is described as unstable; and if $[\lambda_i] < 1$, then with $x(0) \neq 0$, the sequence $x(t)_{t=0}^{\infty} \to 0$ and the system is described as stable. However, if $[\lambda_1] = 1$ and $[\lambda_i] < 1$ for $i \neq 1$, then $\lim_{t\to\infty} x(t) = b_1 u_1$. Under the latter conditions, $\lim_{t\to\infty} x(t)$ represents a steady state or equilibrium for the process modelled by equation (24).

Regarding the eigenvalues reasonably expected from $[I - [(M(1 + g_x(t))]^{-1}$, one can consider the behaviour of the augmented matrix $(M(1 + g_x(t))$. Since, M has a dominant eigenvalue $0 < \frac{1}{(1+g_x(t)} < 1$, it must hold that, $M(1 + g_x(t))$ has an eigenvalue of 1. Further, $[I - (M(1 + g_x(t))]x = 0$. It follows that $\frac{(1+g_x(t))}{g_x(t)}$ is an eigenvalue of $[I - [(M + g_x(t)B)]^{-1}$. In general, we can use the Rayleigh Quotient to estimate,

$$29.\ \mu = \frac{1}{(1-\lambda)} = \frac{x^*[I-[(M(1+g_x(t))]^{-1}x}{x^*x} > 1$$

Now, if $\frac{1}{(1-\lambda)}$ is an eigenvalue of $[I - [(M + g_x(t)B)]^{-1}$, then for $g_x(t)$ a constant, $\frac{(1+g_x(t))\alpha}{(1-\lambda)}$ is an eigenvalue of A. As before, by equation (8), $g_x(t) > 0$ defines a balanced growth path, though its stability is quite another matter. The main parameters dictating long term evolution are α, the key indicator of the allocation of output between current final demand and intermediate and final investment, and λ, the key indicator of economic productivity. If α and λ are sufficiently small, then it is possible that $\frac{(1+g_x(t))\alpha}{(1-\lambda)} < 1$ and the sequence $x(t)_{t=0}^{\infty} \to 0$. However, at any time t it is also possible that α and λ are such that $\frac{(1+g_x(t))\alpha}{(1-\lambda)} > 1$ or $\frac{(1+g_x(t))\alpha}{(1-\lambda)} = 1$, producing instability or a steady state. Thus, much depends on the share of intermediate consumption and investment in total output as well as their productivity. The smaller is α and λ, the greater the prospect for stable growth.

ON STRUCTURAL CHANGE

So far, it has been assumed that $[I - (M + g_x(t)B)]^{-1} > 0$, which also implies that the Hawkins-Simon condition (Hawkins and Simon, 1949) must hold. This condition indicates that if $M + g_x(t)B$ is irreducible, then all the principal minors of $(I - (M + g_x(t)B))$ (and therefore of $(I - (M + g_x(t)B))^{-1}$), are also positive. So that each subgroup of industries demands less intermediate and final capital inputs from the economy than the value of the output it produces. Moreover, we can assume as did Leontief that M is irreducible by construction from the underlying SUTs. It has also been proved by Schwarz (1966 a, b) and by Berman and Plemmons (1979) that for any square matrix of the same order as M, such as $g_x(t)B$, $M + g_x(t)B$ is also irreducible. This irreducibility is a sufficient condition for the existence of a balance growth path for the system represent by equations (8), (12), (13), (14) (16), (22) and (23). Essentially, in this case it means that each sector of the economy depends on all others, directly or indirectly, for either its current account or some of its capital inputs; and every sector has to deliver its output either directly or indirectly to each of the other sectors, operating as a Sraffian basic commodity. This mechanism of interdependence ensures that all sectors, their capital stocks, and the economy grow at the same rate, by relying on the rate of profit, determination of which provides the additional information needed for determination of $g_x(t)$.

Further, as seen above, there must also exist a maximum eigenvalue $\mu > 1$ and any other eigenvalue is strictly smaller than μ. Associated with this would be a unique positive eigenvector, and all other eigenvectors must have at least one negative or complex element. It follows that if μ is an eigenvalue of $(I - (M + g_x(t)B))^{-1}) > 0$ and generates a positive $x(t)$ with some given $F(t)$, then the positive solution $x(t)$ that is associated with $g_x(t)$ must always evolve on the balanced-growth path defined by equation (28). At worst any arbitrary initial output structure $x(0)$ must evolve in a way that converges to the production structure $x(t)$ associated with $g_x(t)$ in equation (28). This assures a meaningful trajectory for any initial $x(0)$.

It might be argued that the solution offered by (12) with a balanced growth rate across all sectors compares favourably to real life goal-oriented competition processes, which cause convergence of the growth rates of different sectors to a unifying competitive equilibrium growth rate. Such technologically determined balanced growth, $g_x(t)$, with the capital stocks and all sector outputs growing at the same rate as the economy in any period can only be justified if all industries are equally productive and achieve productivity growth, increased profits, and savings at the same rate; and if there is no structural change, including that driven by the movement of skills from industries with surplus labour to industries experiencing labour shortages. However, to be applicable to the development process of Caribbean economies, which is focused heavily on structural change, it cannot be presumed that $g_x(t)$ is identical for all industries. The specification of $g_x(t)$ must allow for structural change in that, whatever growth is presumed, whether positive or negative, it must not necessarily be the same for individual industries and for the economy as a whole. Different industries must be able to contract, maintain their capacity, or expand simultaneously with the economy. In particular, it must be possible for $g_x(t)$ to be given the interpretation of a weighted average across all the industries of the economy.

The dynamic model set out so far can be used to capture the effects of technical and institutional change that support progression on a path that allows representation of change in the structure of the economy. The assumption that M, B, L and Z are time invariant is not admissible when markets, other institutions and technology, and the structure of the economy, change continuously, especially as domestic industries are incentivised and learn to produce and export capital. In that context, existing firms may introduce innovations into their technical production processes as a result of learning and problem-solving, and foreign technical change. Firms might enter and exit an industry causing changes of methods, new methods may be adopted from abroad or new products introduced. The changes might lead to productivity growth of some factors. Moreover, some or all of these adjustments might be in response to changes in output or input prices, trade opportunities. The reality of these changes is recognized in the emphasis of UN (2018: 472) that "it is highly important to capture rapid changes in the economy within quarterly periods (or annual periods) which may go unnoticed in the annual or five-yearly structural statistics. For example, with the impact of globalization, the advent of new industries and products, rapid technological change, and other developments, it is recommended that data on sales and purchases are collected more regularly through business surveys. This will ensure that structural change is picked up quickly … Even such traditional industries as electricity, gas, oil and the like change their input structures rapidly." Moreover, it cannot be expected that all industries will adopt the same pattern of adjustment of their input structures, especially their labour and capital requirements, even in a content of intensifying market competition.

Representation of this bewildering variety of changes and causes is not a simple matter, whatever the framework of analysis. If, nevertheless, some technological or institutional innovation is allowed in, then a simple proposition is that the coefficient matrix must carry a time signature allowing some of the elements of $M + g_x(t)B$ to become smaller. In that case, the maximum λ decreases and therefore so does $g_x(t)$. Correspondingly, at least one factor productivity grows, and the available surplus to pay factors also grows. In a context of a given wage rate, the rate of profit and savings would also grow, especially if import coefficients fall, enabling either s_p or $r(t)$ to fall. In the framework developed so far, structural decomposition analysis can used to identify the sources of structural change in the economy. This is done by empirically and analytically tracing changes in the parameters of the model, including $M + g_x(t)B$, following suggestions by authors such as Skolka (1977), Rose and Chen (1991) and Rose and Casler (1996).

Using equation (12), consider the production structure of the economy in two successive periods:

30. $x(t) = [I - (M(t) + g_x(t)B(t))]^{-1}F(t)$

And

31. $x(t+1) = [I - (M(t+1) + g_x(t+1)B(t+1))]^{-1}F(t+1)$

Then, following Skolka (1977; 1989), we can write $dx = x(t+1) - x(t)$ to represent changes in the structure of the economy, since by the definition of a vector the change is calculated element by element allowing for different rates and directions of change of the industries. Therefore,

32. $dx = [I - (M(t+1) + g_x(t+1)B(t+1))]^{-1}F(t+1) - [I - (M(t) + g_x(t)B(t))]^{-1}F(t)$

Adding and subtracting $[I - (M(t+1) + g_x(t+1)B(t+1))]^{-1}F(t)$ in equation (32) gives the decomposition:

$$33.\ dx = \left\{\left[I - \left(M(t+1) + g_x(t+1)B(t+1)\right)\right]^{-1} - \left[I - \left(M(t) + g_x(t)B(t)\right)\right]^{-1}\right\}F(t) \\ +[I - (M(t+1) + g_x(t+1)B(t+1))]^{-1}(F(t+1) - F(t))$$

In equation (33), the change in the structure of production is represented by: (i) change in the total requirements matrix over the period, multiplied by the final demand of the base year, $F(t)$; and (ii) change in final demand over the period multiplied by the total requirements matrix of the current year, $[I - (M(t+1) + g_x(t+1)B(t+1))]^{-1}$(Chenery, *et al*,1962; Kubo and Robinson, 1984). It is clear from the decomposition that the change in the total requirements matrix, and hence structural change, over the period depends on the rates of growth $g_x(t+1)$ and $g_x(t)$. This dependence is preserved in any alternative decomposition.

For example, adding and subtracting $[I - (M(t+1) + g_x(t+1)B(t+1))]^{-1}F(t+1)$ in equation (32) instead gives the alternative decomposition:

$$34.\ dx = \{[I - (M(t+1) + g_x(t+1)B(t+1))]^{-1} - [I - (M(t) + g_x(t)B(t))]^{-1}\}F(t+1) \\ +[I - (M(t) + g_x(t)B(t))]^{-1}(F(t+1) - F(t))$$

In the case of equation (34), the rate of structural change depends on: (i) the rate of change of the total requirements matrix over the period multiplied by final demand in the current period, $F(t+1)$; and (ii) change in final demand over the period multiplied by the total requirements matrix of the base year, $[I - (M(t) + g_x(t)B(t))]^{-1}$, as in Nijhowne, *et al* (1984) and Rose and Chen (1991). The dependence of the rate of structural change on $g_x(t+1)$ and $g_x(t)$ is preserved.

Equations (33) and (34) are mathematically equivalent analogues of the continuous matrix time derivative of equation (12) but the resulting dx will differ because of the different reference periods that transmit the impact of technical change as well as the different total requirements matrices that transmit the impact of change in effective demand. It was suggested by Dietzenbacher and Los (1998) that the arithmetic mean of equations (33) and (34) provides a valid unique estimate because it applies midpoint weights to the changes. Let $T(t)$ be the total requirements matrix of period t. Then, the suggested strategy gives:

$$35.\ dx = dT\frac{[F(t+1)+F(t)]}{2} + \frac{[T(t+1)+T(t)]}{2}dF$$

In equation (35), $dT\frac{[F(t+1)+F(t)]}{2}$ measures the rate of institutional and technical change and $\frac{[T(t+1)+T(t)]}{2}dF$ measures the rate of change of effective demand. Since both terms depend on $g_x(t+1)$ and $g_x(t)$, the result emphasizes the dependence of the rate of structural change on the rate of growth in each period, with the rate of growth being an endogenous variable that is explained by $r(t)$. Thus, to identify the rate of structural change (dx) in either case, it is necessary to explain $g_x(t)$ in terms of $r(t)$ for any t using additional information not represented in equation (35).

UNBALANCED GROWTH AND THE PLANTATION ECONOMY

To fully represent the concerns of Best (1968; 1975) that the economy develops through a process of unbalanced growth in which the cluster of residentiary industries grow faster than the cluster of foreign capital- and import-dependent industries, it is necessary to resort to an appropriate partitioning of the inter-industry matrix. Let

36. $V(t) = (M(t) + g_x(t)B(t)).$

We assume that, after suitable reclassification and permutation, $V(t)$ can be partitioned consistent with Best (1968) into an indecomposable residentiary submatrix V_R, an indecomposable plantation submatrix V_p and a submatrix H that represents the direct interindustry relations between the residentiary industries and the plantation industries. That is,

37. $V(t) = \begin{bmatrix} V_R & H \\ 0 & V_P \end{bmatrix}$

There are two cases.

CASE 1: CLOSED SECTORS

In this case, we assume that the two broad sectors are closed. So, Best's (1968) residentiary sector sells nothing, including capital, to the plantation sector and $H = 0$. In that case, since V_R and V_P are indecomposable, and each has a unique maximum eigenvalue, $\lambda_R < 1$ and $\lambda_p < 1$, respectively, then eventually each will achieve its own path of balanced growth, with sequences governed by $\frac{(1+g_{xR}(t))\alpha_R}{(1-\lambda_R)}$ and $\frac{(1+g_{xP}(t))\alpha_P}{(1-\lambda_P)}$, respectively, even if its initial output vector is arbitrary. Specifically, the rate of balanced growth of the sector defined by V_R, i.e., $g_{xR}(t)$, is not the same as the rate of balanced growth, $g_{xP}(t)$, in the sector defined by V_P. Thus, there is balanced growth within the sectors but unbalanced growth in the economy as a whole. The overall rate of growth of the system would be defined by a weighted average of the two balanced growth rates, with the weights defined by the relative size of the output produced by each sector.

CASE 2: SECTOR LINKAGE THROUGH DEVELOPMENT OF RESIDENTIARY PRODUCTION

In this case, assume that at least some elements of $H \neq 0$. This could be achieved through appropriate policy interventions that stimulate residentiary production and supply of capital to the plantation sector. Then, there must exist some columns of V_P whose elements have become smaller as a result of the use of supplies from the residentiary sector. Thus, such a decline in coefficients would also cause the maximum eigenvalue of V_P to fall. Moreover, if the influence of the residentiary sector on the planation sector grows enough to cause the column sum of V_P to fall sufficiently, then, for given α_P, it becomes possible that $\lambda_p < 1$ falls sufficiently until $\frac{(1+g_{xP}(t))\alpha_P}{(1-\lambda_P)} < 1$ and in the long run the output vector of the plantation sector, x_P, will converge to zero. For example, this is possible if international demand for export staple stimulates low sector growth, say at 1.5% annually, value-added to generate surplus for repatriation is modest, at say 0.4, and most of the output (say 55%) is exported or used for public and private consumption. On the other hand, if the result of policy interventions yields λ_R and α_R such that $\frac{(1+g_{xR}(t))\alpha_R}{(1-\lambda_R)} \geq 1$ then the output vector x_R will either achieve a steady state or grow continuously. For example, if policy stimulates residentiary sector growth at say 5% annually, intermediate and final capital are sufficiently productive to generate value-added to pay wages, taxes, and profits of about 50% of output, so $\lambda_R = .5$, and residentiary exports are such that more than half of output is allocated to final demand other than investment, then $\frac{(1+g_{xR}(t))\alpha_R}{(1-\lambda_R)} > 1$ and the residentiary sector will expand continuously. Further, since $\frac{(1+g_{xR}(t))\alpha_R}{(1-\lambda_R)} > \frac{(1+g_{xP}(t))\alpha_P}{(1-\lambda_P)}$, the two sectors will grow in an unbalanced way and the residentiary sector will eventually take over the economy, up to some limit such as might be set by the necessity of a plantation sector to earn foreign exchange to cover necessary imports.

EXPLAINING UNBALANCED GROWTH

The dynamic model above is consistent, with a technologically determined stable growth path influenced by $g_x(t)$, which also makes $x(t)$ and $p(t)$ feasible from the point of view of behaviour with respect to installed capacity and supply of primary factor (value-added) requirements. However, it leaves two key matters unaddressed. One is that, for any arbitrary time path of investment demand, it provides no explanation of how validating savings are brought in line to guarantee that the investment can be sustained over time. The other is that, for an import-dependent economy such as exists in the Caribbean, it provides no mechanism by which exports are adjusted to guarantee coverage of the imports necessary to sustain the investment over time consistent with positive values of $x(t)$. Another way to view this is through the Brauer-Solow condition. That is, mechanisms must exist to enable assertion with confidence that if M is constructed using monetary values, then in each sector, the money left over to cover or validate financing of investment and imported inputs is always positive. The following explanations amount to proof that some choice of units exist such that all column sums of $M + Z$ are smaller than unity and also, $1 - (m_{ij} + g_x(t)b_{ij}) \geq F_i$.

SAVINGS AND INVESTMENT

Under Caribbean conditions, the first essential task is to specify the relationship between savings growth and investment in the subsystems and thus the information needed to identify $g_x(t)$ for each subsystem. For this, we return to the characteristic conditions that for each of the subsystems it would hold that:

38. $S_R = s_{\pi R} r_R K_R$

39. $S_p = s_{\pi P} r_p K_p$

where S_R and S_p are the gross savings of the residentiary sector and plantation sector, respectively, corresponding to which are the savings rates from profits, $s_{\pi R}$ and $s_{\pi P}$, the profit rates, r_R and r_p, and the stocks of capital, K_R and K_p. Then, from (38) and (39), and the assumption that savings validate investment, it would also hold that:

40. $g_{xR}(t) = s_{\pi R} r_R(t)$
41. $g_{xP}(t) = s_{\pi P} r_P(t)$

Now, assume that $s_{\pi R}$ and $s_{\pi P}$ are set exogenously. Then, the challenge is to identify a relevant principle for determination of the free variables $r_R(t)$ and $r_P(t)$ using applicable facts. Here, it is appropriate to follow the Lewis (1954) observation that savings depend on the rate of operating surplus, which depends on the nexus of production of capital, investment, and the adjustment of technology, productivity, prices, and the wage rate in the sectors of the economy, all without setting up an explosive process. In such circumstances, $r_R(t)$ and $r_P(t)$ can differ and lead to different sector rates of growth.

For the plantation sector, most final capital is imported and used to employ the labour power of local workers and managers, with little reliance on their knowledge, skills, and self-confidence. Sector institutions that adjust to worker resistance over time ($h_p(t)$) also underlie the stock of capital accumulated (Best, 1968). So, real output can be described by a composite function, $Y_p = Y_p(N_p(K_p(h_p(t))))$, where $N_p(K_p(h_p(t)))$ is work effort. The knowledge, skills, and self-confidence on which the sector relies is imported and included in K, so the applicable income identity is:

42. $r_p K_p(h_p(t)) = p_p Y_p - w_p N_p$

where p_p is exogenously determined output price and w_p is the subsistence wage plus relocation premium at which labour supply is available from the under-capitalised and hence employed labour in the residentiary sector. Using the total differential, the identity in (42) yields:

43. $r_p = \frac{1}{\left(1+\frac{K_p}{r_p}\frac{dr_p}{dK_p}\right)}\left[\left(p_p\frac{dY_p}{dN} - w_p\right) + \left(p_p y_N \frac{Ndp_p}{p_p dN} - w_p \frac{Ndw_p}{w_p dN}\right)\right]\frac{dN}{dK_p}\frac{dK_p}{dh_p}\frac{dh_p}{dt}$

where $y_N = \frac{Y}{N}$ is the average productivity of labour in the sector, and it is assumed that plantation institutions are adjusting to worker resistance over some learning time, t. Now, p_p and w_p are determined independent of N. In particular, p_p is determined in the foreign economy and the plantations have no market power, so $\frac{Ndp_p}{p_p dN}$ is just an observable elasticity with p_p subject to random shocks. On the other hand, workers have no labour market power because of surplus labour, so $\frac{Ndw_p}{w_p dN} = 0$. In addition, productivity decomposes into exports per worker x_{pN} and per worker output produced to satisfy domestic demand y_{pN}^d, with the latter including infrastructure and other intermediate supplies. Thus, equation (43) reduces to:

$$44.\ r_p = \frac{\frac{dN}{dK_p}\frac{dK_p}{dh_p}\frac{dh_p}{dt}}{\left(1+\frac{K_p}{r_p}\frac{dr_p}{dK_p}\right)}\left[\left(p_p\frac{dY_p}{dN} - w_p\right) + p_p(x_{pN} + y_{pN}^d)\frac{Ndp_p}{p_p dN}\right]$$

Equation (44) says that the plantation sector's rate of profit (operating surplus) depends on the job-creating tendencies of sector investment, $\frac{dN}{dK_p}\frac{dK_p}{dh_p}\frac{dh_p}{dt}$, on the difference between the marginal product of labour and the wage rate, $\left(p_p\frac{dY_p}{dN} - w_p\right)$, as well as on labour productivity, $y_N = x_{pN} + y_{pN}^d$, and on the elasticity of price with respect to employment, $\frac{Ndp_p}{p_p dN}$.

Now, to understand how y_{pN}^d influences the rate of profit, one must bring imported inputs into the picture. Imports are factor inputs into production, used either directly as capital inputs or indirectly as consumer supplies used to reproduce the labour force. Thus, we turn to the sector's aggregate supply identity $Y_p^A = y_{pN}^d N + \frac{\varepsilon p_m}{p_p} M_p$, where $\frac{\varepsilon p_m}{p_p}$ is the sector's real exchange rate with εp_m the price of imported inputs in domestic currency units and M_P the imported inputs used in current production by the sector. Here, we can write:

$$45.\ y_{pN}^d N = Y_p^A - \frac{\varepsilon p_m}{p_p} M_p$$

Or, using the total differential of (45),

$$46.\ y_{pN}^d = \frac{dM_P/dN_P}{\left(1+\frac{N_P dy_{pN}^d}{y_{pN}^d dN_P}\right)}\left[\left(\frac{dY_p^A}{dM_P} - \frac{\varepsilon p_m}{p_p}\right) + \frac{\varepsilon p_m}{p_p}\left(\frac{M_P}{p_p}\frac{dp_p}{dM_P} - \left(\frac{M_P dp_m}{p_m dM_P} + \frac{M_P d\varepsilon}{\varepsilon dM_P}\right)\right)\right]$$

Equation (46) indicates that the sector's average product of labour in activity designed to supply domestic demand depends on: (i) the influence of employment growth on imports, $\frac{dM_P}{dN_P}$; (ii) the gap between the increased aggregate supply induced by imports and the real exchange rate of the sector, $\left(\frac{dY_p^A}{dM_P} - \frac{\varepsilon p_m}{p_p}\right)$; and (iii) the gap between the rate of increase of product price and the rate of increase of import costs relative to the rate of increase of imports, $\frac{\varepsilon p_m}{p_p}\left(\frac{M_P}{p_p}\frac{dp_p}{dM_P} - \left(\frac{M_P dp_m}{p_m dM_P} + \frac{M_P d\varepsilon}{\varepsilon dM_P}\right)\right)$. Now, Best (1968) emphasized that both p_p and p_m are subject to random shocks that make $\frac{\varepsilon p_m}{p_p}$ highly volatile. Thus, the size of $\frac{dY_p^A}{dM_P} - \frac{\varepsilon p_m}{p_p}$ and $\frac{\varepsilon p_m}{p_p}\left(\frac{M_P}{p_p}\frac{dp_p}{dM_P} - \left(\frac{M_P dp_m}{p_m dM_P} + \frac{M_P d\varepsilon}{\varepsilon dM_P}\right)\right)$ depends on those shocks.

By Best (1968), a favourable shock to p_p can cause $\frac{\varepsilon p_m}{p_p}$ to be very low, in which case $\left(\frac{dY_p^A}{dM_P} - \frac{\varepsilon p_m}{p_p}\right)$ will be very high. Also, $\left(p_p \frac{dY_p}{dN} - w_p\right)$ will be very high, since the existence of large numbers of underemployed workers will keep down w_p. The favourable price shock will also cause $p_p(x_{pN} + y_{pN}^d)\frac{Ndp_p}{p_p dN}$ and $(\frac{M_P}{p_p}\frac{dp_p}{dM_P} - (\frac{M_P dp_m}{p_m dM_P} + \frac{M_P d\varepsilon}{\varepsilon dM_P}))$ to be high. The overall effect of the price shock will be to boost y_{pN}^d in (46) and the rate of profit of the sector in (44). A high rate of profit will cause rapid increase in sector savings capacity and a rapid growth in the inflow of foreign savings. This will cause rapid growth of capital per worker and exports per worker that, together with the growth of the rate of inflow of foreign capital, will create favourable balance of payments conditions. It will also slow import displacement. This is the "golden age" of Best (1968).

On the other hand, a negative shock to p_p will cause $\frac{\varepsilon p_m}{p_p}$ to be comparatively high, especially if accompanied by rising p_m. In that case, $\left(p_p \frac{dY_p}{dN_p} - w_p\right)$ will fall sharply, along with $p_p(x_{pN} + y_{pN}^d)\frac{Ndp_p}{p_p dN}$. Also $\left(\frac{dY_p^A}{dM_P} - \frac{\varepsilon p_m}{p_p}\right)$ will move closer to zero and also it will hold that $(\frac{M_P}{p_p}\frac{dp_p}{dM_P} - (\frac{M_P dp_m}{p_m dM_P} + \frac{M_P d\varepsilon}{\varepsilon dM_P}))$ will fall sharply. The latter adjustments will drag down labour productivity in equation (46). The joint effects will force down the rate of profit in (44). So, imports will become less affordable and $\frac{dM_P}{dN_P}$ will fall, bringing down $\frac{dY_p}{dN_p}$ relative to w_p and reinforcing the fall in the rate of profit. A low rate of profit will cause rapid decline in sector savings capacity and a rapid decline in the rate of inflow of foreign savings. This will cause a rapid decline in the growth of capital per worker and exports per worker. The fall in foreign capital inflows and exports per worker lead to significant balance of payments challenges and rising indebtedness. The decline in the rate of inflow of foreign savings and exports per worker will also increase import displacement. This is the "gall and wormwood" of Best (1968).

Regarding the sufficiency of sector exports to cover sector imports, it follows from equation (44) and equation (45) that the gap between overall sector productivity and the productivity of labour in supplying domestic demand defines the capacity of sector exports per worker to cover sector imports per worker. That is, to the extent that sector productivity exceeds the productivity of activity aimed at supplying domestic demand, the sector can generate enough exports to cover its import needs:

$$47.\ x_{pN} = \frac{\varepsilon p_m}{p_p} m_{pn} = y_{Np} - y_{pN}^d$$

where $m_{pn} = \frac{M_p}{N_p}$. Foreign capital inflows induced by the rate of profit then supplement that import capacity and show up in the overall balance of payments.

The lengths of the golden age and gall and wormwood are indeterminate, but this kind of volatility and repeat of the history of extended periods of gall and wormwood are significant threats posed by excessive reliance on the externally propelled sector. This is the most compelling warning to Caribbean countries about the importance of structural change.

Matters are different in the residentiary sector. There, much capital is imported though some domestic capital is produced as suggested by Lewis (1954), and work is organised around the produced knowledge, skills, and self-confidence of local workers and managers. Capital is required to employ the knowledge, skills, and self-confidence of workers and managers, and the amount installed depends on the nature of the supporting institutions and the time t over which learning by doing occurs. So real output is best described by a composite function $Y_r = Y(\widetilde{N}(K_r(h_r(t))))$, where $\widetilde{N}(K_r(h_r(t))$ is the input of labour equipped with its knowledge, skills and self-confidence, with $\widetilde{N} = E_n N_r$ for E_n the average level of knowledge, skills and self-confidence of workers and managers, and N_r the number of paid employees. Again, we treat imported direct and indirect factor inputs as included in K_r, so the applicable income identity is:

$$48.\ r_r K_r(h_r(t)) = p_r Y_r - w_r \widetilde{N}_r$$

where p_r is residentiary output price and w_r is the average wage for employee capacities in the sector that includes a sector-specific capacity premium over its subsistence wage. Using the total differential, the identity in (48) yields:

$$49.\ r_r = \frac{\frac{dN}{dK_r}\frac{dK_r}{dh_r}\frac{dh_r}{dt}}{\left(1+\frac{K_r}{r_r}\frac{dr_r}{dK_r}\right)}\left[\left(p_r\frac{dY_r}{d\widetilde{N}_r} - w_r\right) + \left(p_r(x_{\widetilde{N}r} + y^d_{\widetilde{N}r})\frac{\widetilde{N}_r dp_r}{p_r d\widetilde{N}_r} - w_r\frac{\widetilde{N}_r dw_r}{w_r d\widetilde{N}_r}\right)\right]$$

where $\left(x_{\widetilde{N}r} + y^d_{\widetilde{N}r}\right) = y_{\widetilde{N}r} = \frac{Y_r}{\widetilde{N}_r}$ is the average productivity of knowledge, skills and self-confidence in the residentiary sector. Here too, $y_{\widetilde{N}r}$ decomposes into exports per worker ($x_{\widetilde{N}r}$) and per worker supply of output designed to satisfy domestic demand ($y^d_{\widetilde{N}r}$). Equation (49) says that the subsector's rate of profit depends on three factors: (i) the rate of increase in employment of the knowledge, skills, and self-confidence of workers in the sector under the influence of institutional development over learning time ($\frac{dN}{dK_r}\frac{dK_r}{dh_r}\frac{dh_r}{dt}$); (ii) the difference between the marginal product of labour and the corresponding wage rate $\left(p_r\frac{dY_r}{d\widetilde{N}_r} - w_r\right)$; and (iii) the term $(p_r(x_{\widetilde{N}r} + y^d_{\widetilde{N}r})\frac{\widetilde{N}_r dp_r}{p_r d\widetilde{N}_r} - w_r\frac{\widetilde{N}_r dw_r}{w_r d\widetilde{N}_r})$, which describes the difference between the average product of knowledge, skills and self-confidence and the wage, respectively adjusted by the elasticities of price and the wage with respect to the knowledge, skills and self-confidence of workers and managers. The price elasticity $\frac{\widetilde{N}_r dp_r}{p_r d\widetilde{N}_r} \neq 0$ reflects the extent of market power enjoyed by producers in the product markets pursued by residentiary producers, which are typically characterised by monopolistic competition. The wage elasticity $\frac{\widetilde{N}_r dw_r}{w_r d\widetilde{N}_r} \neq 0$ reflects the extent of market power in the labour market enjoyed by labour because of its knowledge, skills, and self-confidence.

To understand how $y^d_{\widetilde{N}r}$ influences the sector's rate of profit, it is again necessary to bring imported inputs into the picture. Here too, imports are factor inputs into production, used either directly as capital inputs or indirectly as consumer supplies used to reproduce the labour force. Thus, we turn to the aggregate supply identity $Y_r^A = y^d_{\widetilde{N}r}\widetilde{N}_r + \frac{\varepsilon p_m}{p_r} M_r$, where $\frac{\varepsilon p_m}{p_r}$ is the sector's real exchange rate with εp_m the price of imported inputs in domestic currency units and M_r the imported inputs used in current production by the sector. Then:

$$50.\ y^d_{\widetilde{N}r}\widetilde{N}_r = Y_r^A - \frac{\varepsilon p_m}{p_r} M_r$$

Or, using the total differential of (50),

$$51.\ y^d_{\widetilde{N}r} = \frac{dM_r/d\widetilde{N}_r}{(1+\frac{\widetilde{N}_r dy^d_{\widetilde{N}r}}{y^d_{\widetilde{N}r} d\widetilde{N}_r})}[(\frac{dY_r^A}{dM_r} - \frac{\varepsilon p_m}{p_r}) + \frac{\varepsilon p_m}{p_r}(\frac{M_r}{p_r}\frac{dp_r}{dM_r} - (\frac{M_r dp_m}{p_r dM_r} + \frac{M_r d\varepsilon}{\varepsilon dM_r}))]$$

Equation (51) indicates that the average product of labour in the generating the sector output that targets domestic demand depends on: (i) the influence of growth of employment of skilled labour on imports, $\frac{dM_r}{d\widetilde{N}_r}$; (ii) the gap between the aggregate supply induced by increasing imports and its real exchange rate, $\frac{dY_r^A}{dM_r} - \frac{\varepsilon p_m}{p_r}$; and (iii) the gap between the rate of increase of product price and the rate of increase of import costs relative to the rate of increase of imports, $\frac{\varepsilon p_m}{p_r}(\frac{M_r}{p_r}\frac{dp_r}{dM_r} - (\frac{M_r dp_m}{p_m dM_r} + \frac{M_r d\varepsilon}{\varepsilon dM_r}))$. The sector's real exchange rate, $\frac{\varepsilon p_m}{p_r}$, is significantly less volatile that its counterpart in the plantation sector, because p_r is not similarly vulnerable to random exogenous shocks but rather reflects domestic price-making capacity.

Here too, the gap between sector productivity and the productivity of activity aimed at supplying domestic demand defines the capacity of the sector to generate enough exports to cover its import needs. That is:

$$52.\ x_{rN} = \frac{\varepsilon p_m}{p_r} m_{rn} = y_{Nr} - y^d_{Nr}$$

where $m_{rn} = \frac{M_r}{\widetilde{N}_r}$. Again too, foreign capital inflows induced by the rate of profit of the sector then supplement that productivity-based import capacity and show up in the overall balance of payments.

As the sector develops and creates a growing supply of capital, the rate of profit in equation (49) becomes generally higher than that in equation (44), and thus leads to unbalanced growth in the economy, for several reasons. First, $\frac{d\tilde{N}_r}{dK_r}\frac{dK_r}{dh_r}\frac{dh_r}{dt}$ is significantly higher than $\frac{dN}{dK_p}\frac{dK_p}{dh_p}\frac{dh_p}{dt}$, because of the impact of capital accumulation on growth of the knowledge, skills, and self-confidence of workers and because residentiary institutions adjust faster than those of the plantation sector over time. This, the causes $\left(p_r\frac{dY_r}{d\tilde{N}_r} - w_r\right) > \left(p_p\frac{dY_p}{dN} - w_p\right)$. This is reinforced by the assumption of Best (1968) that the accumulation of knowledge, skills and self-confidence in the residentiary sector has no effect on the marginal product of labour in the plantation sector. Second, by Best (1971), the accumulation of knowledge, skills, and self-confidence through capital accumulation in the residentiary sector, $\frac{d\tilde{N}_r}{dK_r}\frac{dK_r}{dh_r}\frac{dh_r}{dt}$, activates an innovation process that introduces an increasing variety of goods and services in residentiary output, which boosts capacity for intra-industry trade with price-making power and causes growth of $\left(p_r(x_{\widetilde{Nr}} + y_{\widetilde{Nr}}^{d})\frac{\tilde{N}_r dp_r}{p_r d\tilde{N}_r} - w_r\frac{\tilde{N}_r dw_r}{w_r d\tilde{N}_r}\right) > 0$. Eventually, subject to policy support from active government, these forces contribute to relatively faster growth of the residentiary sector and to its domination of the economy, as predicted by Best (1968), Best and Levitt (2009), and Best and St Cyr (2012).

SUMMARY

Notwithstanding its innovative representation of the historical evolution of the plantation economy, the static Leontief framework adopted by Best (1968) and Best and Levitt (1969) is open. The key final demand vectors of investment, consumption, government spending and exports are all exogenous. In that case, exogenous changes in final demand or value-added force responsive changes in output, factor demand, and prices, but there is no specific mechanism by which a change in output could induce a responsive change in the elements of final demand, or by which an adjustment of output price would induce a responsive (or validating) change in factor demand and factor prices, especially the rate of profit. Thus, the plantation economy model cannot adequately represent the process of unbalanced growth that would lead to development in the sense that it intended, and that was also intended by the restatement in Best (1980) and the clarification of intent in Best and Levitt (2009) and Best and St Cyr (2012). This problem can be remedied, at least partially, by employing a dynamic framework, as in Leontief (1953; 1970), in which investment and profits are endogenous.

The key is to construct a unified inter-industry matrix in which each industry is represented as supplying intermediate capital, or final capital, or both, to meet current and future capital stock requirements in the light of existing technology. Also, in the associated price system, profits and savings are identified that validate the investment in expanded capacity. Thus, an industry's current investment is assumed to call for a variety of goods and services to be produced by other industries to add to the former's production capacity with at least one lag relative to the period in which the capacity will be used. The model allows additions to the stocks of durable capital goods to be technologically required, given the technique in use, so that an expansion of productive capacity, validated by the flow of savings from profits, matches the rate of growth of the level of output for which there is effective demand. Further, the model can be represented in standard form using an analytical *total production requirements matrix* that represents the direct and indirect requirements of industry intermediate and final capital output to satisfy a specified level of final uses. It can capture the role of forces such as government policies, the state of the factor markets and related factor prices, product prices, productivity growth and profit growth associated with investment, and consequences for the flow of savings.

The model can also be represented in the form of a standard matrix difference equation for application of theorems related to stability. In particular, assuming the Hawkins-Simon condition holds, then by the Peron-Frobenius theorem on non-negative matrices a dominant eigenvalue exists that is related to the rate of growth and a balanced growth path can be identified which is governed in part by the allocation of output between current final demand and intermediate and final investment, and by the economic productivity of intermediate consumption and investment. The smaller the share of current final demand in output and the greater the productivity of resource use, the greater the prospect for stable growth.

An appropriate matrix partition was identified that allows representation of the economy in terms of the plantation subsystem and the residentiary subsystem, with linkage created by the evolution of the residentiary system. In that partition, the economy develops through a process of unbalanced growth in which the cluster of residentiary industries grow faster than the cluster of foreign capital- and import-dependent industries. This is explained by the condition that the growth rate of the residentiary subsystem is boosted by a rising rate of profit tied to innovation-led labour productivity growth relative to the wage rate and by a rising rate of exports tied to similar marginal import efficiency growth relative to the real exchange rate. Such dynamics are not as pervasive in the plantation subsystem where the rate of domestic savings out of profit is also lower than in the residentiary subsystem, partly because of the repatriation of profits and partly because the rate of profit of the plantation subsystem tends to be reduced over time by uncertainty and related extended periods of stagnation.

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